Aggregate planning

- **Aggregate planning** : is concerned with determining the quantity and timing of production for the intermediate future, often from 3 to 18 months.

هي عملية تحديد حجم وميعاد الإنتاج لفترة زمنية متوسطة تتراوح بين
٣ إلى ١٨ شهر

- Operation managers try to determine the best way to meet forecasted demand by adjusting:

العوامل التي تعتمد عليها تلبية الطلبات من الإنتاج

1. production rates
2. labor levels
3. Inventory levels
4. Overtime work
5. Subcontracting rates

- Objective of aggregate planning is to meet forecasted demand while minimizing cost over the planning period

- Costs :

1- Production Costs :

- material
- Labor
  - over time
  - Sub Contracting

2- Inventory Costs :

- holding
- Shortage (back order)

3- Capacity Change

- hiring and training workers
  تأجير عمال لزياده الأنتاج
- lays off workers (firing)
  طرد عمال عند أنخفاض الطلب وذلك لعدم حاجة المصنع لهم

Spreadsheet methods

1_ Zero inventory plan (lot for lot)

produce exactly the demand for each period, which requires a fluctuating workforce , no inventory is carried

يتم أنتاج المطلوب من كل فتره بدون زياده وذلك يتطلب تغيير عدد العمال

أعداداً على حجم الأنتاج المطلوب , مما يترتب عنه عدم وجود منتجات من المخزن

Workers are added when demand increases and are level off when demand decreases.

Example2_1:

X_Print Manufacturing makes laser printers. One plant assembles the PL_4000 model. Standards indicate that one worker can assemble five printers per day. This model costs

about $350 to make, and the company figures it costs $5 to hold one printer in inventory for one month. Workers earn $1500 per month and can be hired for $500 each; firing a worker costs $750. Currently, there are 12 workers in the assembly department. If a printer is backordered, the cost is $35 per unit per month.

Month	July	Aug	Sep	Oct	Nov	Dec	total
working days	21	22	21	23	19	20	126
Demand	1020	950	800	1000	1250	650	5670

Given:

1. Capacity (worker productivity)

production = 5 unit / worker / day

2_ Costs:

making Cost = $ 350 / u

holding Cost = $ 5 / u / month

الرواتب
Wages = $ 1500 / worker / month

hiring Cost = $500 / worker

firing Cost = $ 750 / worker

shortage Cost = $35 / u / month
(back order)

3_ Number of existing labours :

Currently 12 workers

Solution:

		Jul	Aug	Sep	Oct	Nov	Dec	Total
1	Days	21	22	21	23	19	20	126
2	u/worker	105	110	105	115	95	100	630
3	Demand	1020	950	800	1000	1250	650	5670
4	Workers needed	10	9	8	9	14	7	57
5	" available	12	10	9	8	9	14	-
6	" hired	0	0	0	1	5	0	6
7	Hiring Cost	0	0	0	500	2500	0	3000
8	Worker laid off	2	1	1	0	0	7	11
9	lay off cost	1500	750	750	0	0	5250	8250
10	Workers used	10	9	8	9	14	7	57
11	labor Cost	15000	13500	12000	13500	21000	10500	85500
12	units produced	1020	950	800	1000	1250	650	5670
13	Net inventory	0	0	0	0	0	0	0
14	Holding Cost	0	0	0	0	0	0	0
15	BackOrder Cost	0	0	0	0	0	0	0
16	Total Cost	16500	14250	12750	14000	23500	15750	96750

making Cost = 350 * total units produced

= 350 * 5670 = 1,984,500

Total Cost = 1,984,500 + 96750 = 2,081,250

Rows description:

1- No. of days / month : given

2- No. of units produced by a worker per month :

② = ① * unit/worker/day

3- Demand : given

4- Workers needed = $\frac{\text{Demand}}{\text{u/worker}}$

④ = $\frac{③}{②}$ تقرب لأعلى

5- Workers available :

- put the no. of existing workers (given) in the first month

أكتب عدد العمال الموجود من المصنع (المعطى) في أول شهر

- and write down the no. of row ④

وبعد ذلك يتم كتابة أداد العمال من الصف ٤

6 & 8 -

Workers available - needed = → + Firing
                              → - hiring

⑤ - ④ = → + ⑧
        → - ⑥

7 & 9 -

⑦ = ⑥ * hiring Cost

⑨ = ⑧ * firing Cost

10 - Workers used = workers needed

⑩ = ④

11 - labor Cost = Workers used * Wages

= ⑩ * Wages

12 - Units produced = Workers used * u/worker

= ⑩ * ②

or ⑰ = Demand = ③

13 & 14 & 15 - Zeros (Zero inventory plan)

2- Level work force plan:

level work force plan or Constant work force plan where the same number of workers is used in each period

2.1 Constant production: with backorders

$$\text{no. of workers} = \frac{\Sigma \text{demand}}{\Sigma \text{Capacity}} =$$

av. no. of workers

$$= \frac{1020+950+\cdots+650}{105+110+\cdots+100} = \frac{5670}{630} = 9$$

2.2 Constant production: with no backorders

	Jul	Aug	Sep	Oct	Nov	Dec
① Cap/worker	105	110	105	115	95	100
② = Cum ①	105	105+110=215	320	435	530	630
③ Demand	1020	950	800	1000	1250	650
④ = Cum ③	1020	1970	2770	3770	5020	567
⑤ = ④/②	9.7≈10	10	9	9	10	1

Select max from ⑤ ⟶ no. of workers = 10

Solution of example 2.1

with back orders

		Jul	Aug	Sep	Oct	Nov	Dec	Total
1	Days	21	22	21	23	19	20	126
2	u/worker	105	110	105	115	95	100	630
3	Demand	1020	950	800	1000	1250	650	5670
4	workers needed	9	9	9	9	9	9	54
5	" available	12	9	9	9	9	9	-
6	" hired	0	0	0	0	0	0	0
7	hiring cost	0	0	0	0	0	0	0
8	worker laidoff	3	0	0	0	0	0	0
9	layoff cost ⑧*750	2250	0	0	0	0	0	2250
10	workers used ④	9	9	9	9	9	9	54
11	labor cost ⑩* 1500	13500	13500	13500	13500	13500	13500	81000
12	units produced ⑩*②	945	990	945	1035	855	900	5670
13	Net inventory ⑫-③+⑬ (الى قبلها)	-75	-35	110	145	-250	0	-
14	Holding cost			550	725			1275
15	Backorder cost	2625	1225			8750		12600
16	Total cost	18375	14725	14050	14225	22250	13500	97125

making cost = 1,984,500

Total cost = 1,984,500 + 97125 = 2,081,625

Solution of example 2.1

with no back orders

		Jul	Aug	Sep	Oct	Nov	Dec	Total
1	Days	21	22	21	23	19	20	126
2	$/worker	105	110	105	115	95	100	630
3	Demand	1020	950	800	1000	1250	650	5670
4	workers needed	10	10	10	10	10	10	60
5	" available	12	10	10	10	10	10	-
6	" hired	0	0	0	0	0	0	0
7	hiring cost	0	0	0	0	0	0	0
8	workers laid off	2	0	0	0	0	0	0
9	layoff cost	1500	0	0	0	0	0	1500
10	workers used	10	10	10	10	10	10	60
11	labor cost	15000	15000	15000	15000	15000	15000	90000
12	units produced	1050	1100	1050	1150	950	~~1000~~ 1000-630 → 0	6300
13	Net inventory	30	180	430	580	280	~~630~~ → 0	-
14	Holding cost	150	900	2150	2900	1400	~~3150~~ → 0	10650
15	Backorder cost	0	0	0	0	0	0	0
16	Total Cost	16650	15900	17150	17900	16400	18150	102150

making cost = $(6300)(350) = 2,205,000$ $\quad \Sigma(12)$

Total Cost = 2,307,150

Example 2-2:

Poseidon Meter Inc. Makes a variety of water meters. Data for the past year indicate a worker can make, on average, 100 meter per four week period. The inventory holding cost is computed to be $1 / meter / period. Backorders, if allowed, cost about $ 2 / meter / period. New workers can be hired at a cost of $1000 / worker, and existing workers can be laid off at a cost of $2000 / worker. Workers are paid $1500 / period. There are currently 10 workers at Poseidon. The forecast for the next four periods is 1200, 1200, 1000 and 1000 meters respectively.

Given:

1- Capacity : 100 meter / period

2- Costs:

holding cost = $ 1 / meter / period

backorder cost = $ 2 / meter / period

hiring cost = $ 1000 / worker

laid off cost = $ 2000 / worker

wage = $ 1500 / period / worker

3- No. of existing labors:

Currently 10 workers

1_ Zero Inventory:

		1	2	3	4	total
1	Days					
2	u/worker	100	100	100	100	400
3	Demand	1200	1200	1000	1000	4400
4	workers needed	12	12	10	10	44
5	" available	10	12	12	10	-
6	" hired	2	0	0	0	2
7	hiring Cost	2000	0	0	0	2000
8	workers laid off	0	0	2	0	2
9	layoff Cost	0	0	4000	0	4000
10	workers used	12	12	10	10	44
11	labor Cost	18000	18000	15000	15000	66000
12	units produced	1200	1200	1000	1000	4400
13	net inventory	0	0	0	0	0
14	Holding Cost	0	0	0	0	0
15	Backorder Cost	0	0	0	0	0
16	Total Cost	20000	18000	19000	15000	72000

2- level workforce (with backorders)

$$\text{no. of workers} = \frac{\Sigma \text{Demand}}{\Sigma \text{Capacity}} = \frac{4400}{400} = 11$$

		1	2	3	4	Total
1	days					
2	u/worker	100	100	100	100	400
3	Demand	1200	1200	1000	1000	4400
4	workers needed	11	11	11	11	44
5	" available	10	11	11	11	-
6	" hired	1	0	0	0	1
7	hiring cost	1000	0	0	0	1000
8	workers laid off	0	0	0	0	0
9	laid off cost	0	0	0	0	0
10	workers used	11	11	11	11	44
11	labor cost	16500	16500	16500	16500	66000
12	units produced	1100	1100	1100	1100	4400
13	net inventory	-100	-200	-100	0	-
14	holding cost	0	0	0	0	0
15	backorder cost	200	400	200	0	800
16	total	17700	16900	16700	16500	67800

3. level workforce : (no. backorders)

Cum demand	1200	2400	3400	4400
Cum Capacity	100	200	300	400
no. of workers	12	12	12	11

		1	2	3	4	total
1	days					
2	u/worker	100	100	100	100	400
3	demand	1200	1200	1000	1000	4400
4	workers needed	12	12	12	12	48
5	" available	10	12	12	12	-
6	" hired	2	0	0	0	2
7	hiring cost	2000	0	0	0	2000
8	workers laid off	0	0	0	0	0
9	laid off cost	0	0	0	0	0
10	workers used	12	12	12	~~12~~ 8	40 ~~48~~
11	labor cost	18000	18000	18000	18000	72000
12	units produced	1200	1200	1200	~~1200~~ -400 800	
13	net inventory	0	0	200	~~400~~ 0	200
14	holding cost	0	0	200	0	200
15	backorder cost	0	0	0	0	0
16	total cost	20000	18000	18200	18000	74200

*Linear programming approaches to aggregate planning:

objective function:

$$\text{Minimize} \sum_{t=1}^{T} \text{Costs}$$

(making) (wages) (hiring & laying off)

$$\sum \left[C_t^P P_t\right] + \left[C_t^W W_t\right] + \left[C_t^H H_t + C_t^L L_t\right]$$

$$+ \left[C_t^I I_t + C_t^B B_t\right]$$

(inventory & backorder)

where:

- P_t : # of units produced in period t
- C_t^P : Cost to produce one unit in period t

- W_t : # of workers available
- C_t^W : Cost of one worker

- H_t : # of workers hired
- C_t^H : Cost to hire one worker
- L_t : # of workers laid off
- C_t^L : Cost to layoff one worker

- I_t : # of units held in inventory at the end of period t
- C_t^I : Cost to hold one unit in inventory for period t
- B_t : # of units backordered at the end of period t
- C_t^B : Cost to backorder one unit for period t

Subject to

Production

1. *Capacity Constraints

$$P_t \leq n_t W_t \qquad t = 1, 2, \ldots, T$$

n_t : worker productivity in period t

2. Work Force Constraints

$$W_t = W_{t-1} + H_t - L_t \qquad t = 1, 2, \ldots T$$

inventory blance

3. ~~Material~~ Constraints (inventory)

net inventory this period = net inventory last period + production this period − demand this period

$$[I_t - B_t] = [I_{t-1} - B_{t-1}] + P_t - D_t$$

$$P_t, W_t, H_t, L_t, I_t \geq 0$$